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11. SUPPLEMENTARY NOTES

The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

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13. ABSTRACT (Maximum 200 words)

In completed research: (a) We have tested, with positive results, the Ansatz that is used in the higher order Lax-Levermore theory developed by us (with T. Zhang). (b) We have shown the stability of solitary pulses through a semiconduction in the Gunn effect (with L.L. Bonilla).

In on-going research: (a) we have established strong results (analytically and numerically) in the semifinite Toda chain with time-dependent (periodic) forcing. Our work here goes beyond integrable theory (with P. Deift and T. Kricherbauer). (b) We have observed numerically that certain spectra in nonintegrable cases remain fixed in some averaged sense (with M. McDonald). (c) We have made some progress in the long-standing problem of the initial-boundary value problem for the KdV equation (with A. Fokas).

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DISPERSIVE REGULARIZATION OF SHOCKS

Final Report

STEPHANOS VENAKIDES

August 25, 1992

U. S. ARMY RESEARCH OFFICE

GRANT NUMBER: DAA L03-91-G-0178

DUKE UNIVERSITY

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A. Problems and Results.

1. *Periodic Limit of Inverse Scattering* with T. Zhang.

In previous work, we have extended the Lax-Levermore calculation of the zero-dispersion limit for the Korteweg-de Vries equation to a higher order. Based on an Ansatz involving a quantum condition we have recovered not only the weak limit of the solution, hut also the oscillatory small-scale structure. One of the purposes of the present work is to verify the correctness of the Ansatz in the higher order theory. We introduce a known spatial oscillation in the form of a periodic potential and then we recover it using the machinery of the higher order theory. The results are very satisfying. It is not only the wave numhers, frequencies and the mean value that are correctly recovered hut also the phase-shifts which the higher order theory does not claim to be able to recover. The calculation of the phase shifts is quite delicate. Of course the scattering data of our initial oscillation are exactly known, whereas in the case of general initial data, the scattering data are known only up to WKB/turning point approximations. Thus, hased on our present work we do not yet claim to he ahle to derive the phase-shifts of the general prohlem correctly. However, our result seems to suggest that for general initial data the phase-shifts resulting from the higher order theory either are correct or can he made correct by more elaborate calculations of the scattering data.

Puhlication:

T. Zhang and S. Venakides, *Periodic Limit of Inverse Scattering*, Comm. Pure Appl. Math., to appear.

2. *The Instability of the Steady-State, and the Stability of the Solitary Wave in $n-GaAs$ Semiconductors (Gunn effect)*, (with L.L. Bonilla)

The Gunn effect is ohserved when a dc voltage is applied through a purely resistive circuit to a semiconductor in which carrier mobility v is a decreasing function of the electric field E over a range of field values. This is known as negative differential resistance and is quantum mechanical in nature. When the value of the dc voltage exceeds a threshold level, a solitary pulse is created at one end of the semiconductor, propagates through and is destroyed at the other end; the phenomenon is then repeated periodically. It is used as the main source of microwave generation.

Results:

(a) We calculate asymptotically (ℓ, the semiconductor length is the large parameter), the instahility threshold for the steady-state by calculating asymptotically the eigenvalues of the linearized prohlem.

(b) We caricature the mobility/electric field curve to he piecewise linear (slope $= +\infty$ at $E = 0$, constant negative slope when $E > 0$). We then show hy the method of characteristics that the linearized prohlem at the solitary pulse is asymptotically stable.

Publication:

L.L. Bonilla and S. Venakides, *On the Stability of the Solitary Wave of the Gunn Effect*, submitted to SIAM J. Appl. Math.

3. *The Toda Shock Problem with Time-Dependent Forcing*, (with P. Deift and T. Kricherbauer).

We examine the piston problem for the semi-infinite Toda chain with an imposed velocity on the zeroth particle (piston) which is time-dependent. The problem is not isospectral. The spectrum of the associated Lax operator varies with time. It initially consists of a single band of spectrum which begins to emit eigenvalues as a result of the forcing.

For a time-periodic forcing, we find from numerical calculation that the emitted eigenvalues tend to cluster into new band structures. In the chain itself, multiphase-waves emerge.

Analytical Results:

(a) We obtain a closed system of equations for the eigenvalues emitted.

(b) We obtain and solve the continuum limit of these equations, thus determining the density of e-values if the position of the bands formed is given. We have not as yet been able to determine analytically the number and position of the bands.

Numerical Results:

(a) The average rate of emission of eigenvalues depends only on the mean value of the driver.

(b) The numer of new bands formed equals one plus the integer part of the fraction:

$$\frac{\text{average period of driver}}{\text{average period of e-value emission}}.$$

(c) The band-gap system generated corresponds to frequencies which are in the frequency module generated by the frequency of the driver and the mean frequency of eigenvalue emission.

Work still in progress.

4. *Newton ball effect in a chain with reflections at fixed end-points.* (with M. McDonald)

We examine a chain of finitely many particles, in which the first and the last particles are fixed. We assume that the chain is initially in equilibrium, except for one particle wbich bas an initial velocity v. We study the cbain analytically by reflecting it about one of the fixed endpoints and tben embedding it into a periodic cbain. When tbe interaction potential is expontial (Toda cbain), tbe system is integrable.

We bave studied tbe evolution of tbe system numerically in the real and in the spectral domain botb in tbe integrable and nonintegrable case. Numerical experiments indicate a localized disturbance wbich travels back and forth tbrougb tbe chain bouncing back at tbe endpoints, combined with an irregular background vibration tbroughout tbe whole cbain.

We form tbe doubly infinite tridiagonal matrix $M(t)$ wbicb represents the state of tbe cbain at time t:

$$M(t) = (\dots\ 0\ b_{n-1}\ a_n\ b_n\ 0\ \dots), \quad n = 0, \pm 1, \pm 2, \dots.$$

b_n and a_n defined by

$$b_n = \frac{1}{2}\sqrt{V(x_{n+1} - x_n)}, \quad a_n = -\frac{\dot{x}_n}{2}$$

where x_n is tbe position of the n^{th} particle. Each of the diagonals of M are spatially periodic with period $N =$ number of particles.

In the integrable case we bave $\dot{M} + MB - BM = 0$. Tbe Lax pair theory implies that $M(t)$ and $M(0)$ are unitarily equivalent; the problem is isospectral. It is remarkable that even in the nonintegrable case, in which $\dot{M} + MB - BM$ equals some complicated expression, tbe spectrum remains fixed in an averaged sense. The endpoints of the spectral bands are on the average constant in time, vibrating irregularly about some fixed value (see illustrations). We observe that in all cases, the spectral bands are distributed along a "macroband" witb the exception of the two outermost bands. This effective band structure explains the localized traveling disturbance.

Work is in progress.

5. *The Emission of Solitons from the Boundary in KdV on the Half-Time with a Boundary Condition at the Origin.*

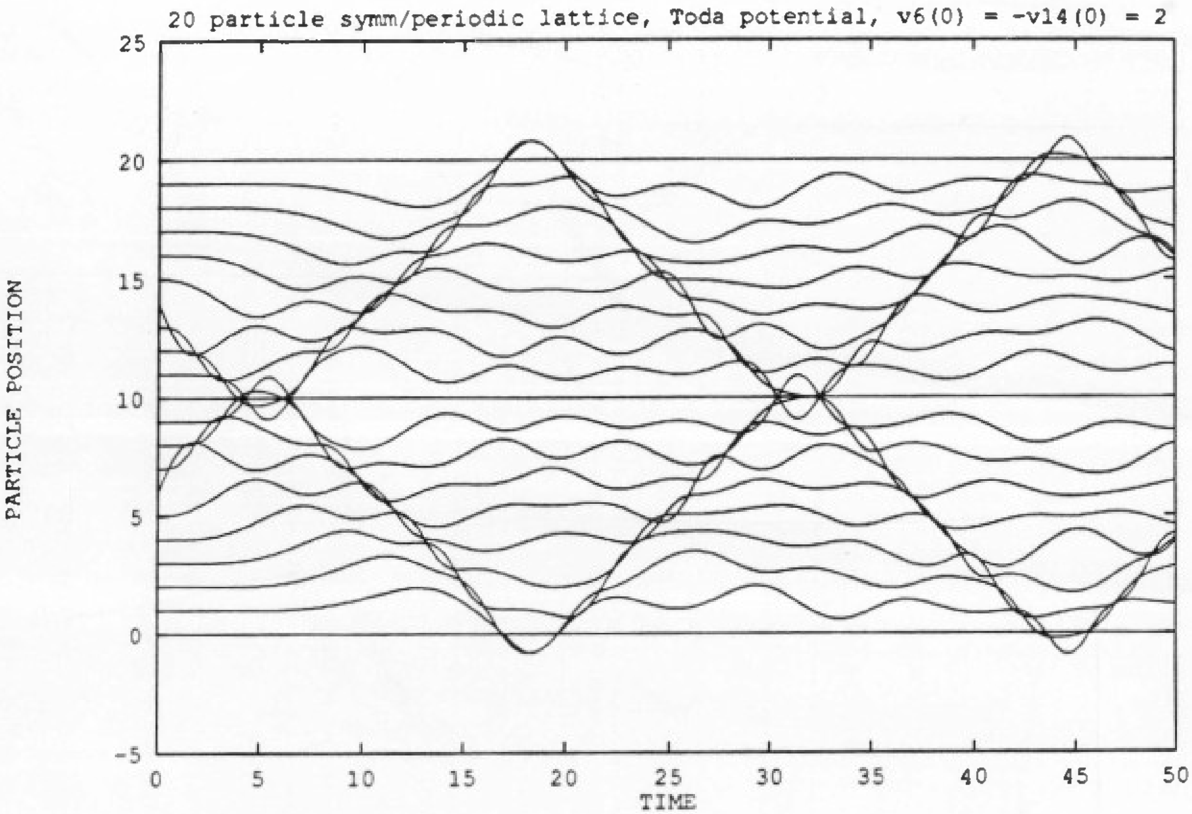
We bave worked with A. Fokas towards understanding this phenomenon. We have establisbed an equation for the evolution of tbe scattering data (this includes varying spectrum) but bave not been able to solve it yet. The main goal is to find tbe correct asymptotic tool so that the frequency of the emission and the amplitude of the soliton pulses can be ascertained from tbe equation. Work in progress.

B. Publications.

1. T. Zhang and S. Venakides, *Periodic Limit of Inverse Scattering*, Comm. Pure Appl. Math., to appear.
2. L.L. Bonilla and S. Venakides, *On the Stability of the Solitary Wave of the Gunn Effect*, submitted to SIAM J. Appl. Math.
3. P.D. Lax, C.D. Levermore and S. Venakides, *The Generation and Propagation of Oscillations in Dispersive IVPs and their Limiting Behavior.* In *Important Developments in Soliton Theory 1980–1990*, T. Fokas and and V.E. Zakharov eds., Springer-Verlag, Berlin 1992.

C. Scientific Personnel.

Stephanos Venakides
Michael McDonald (grad student)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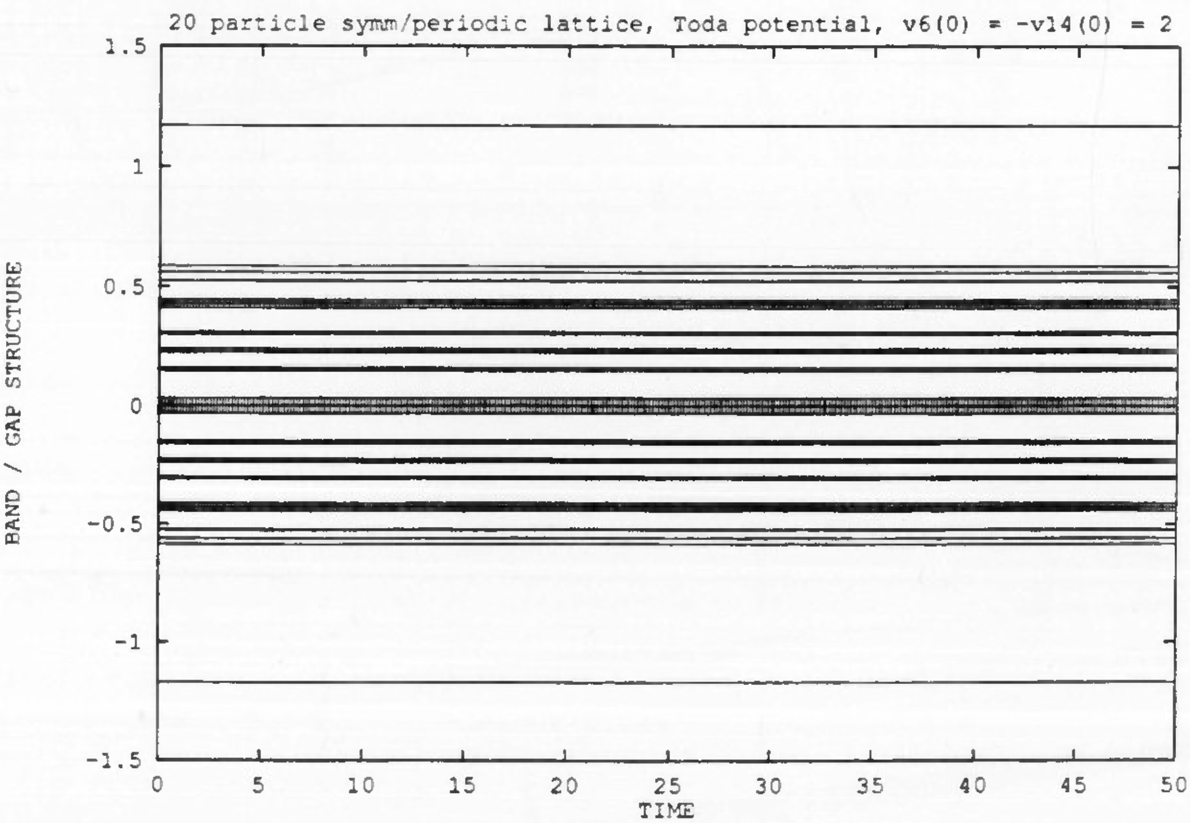
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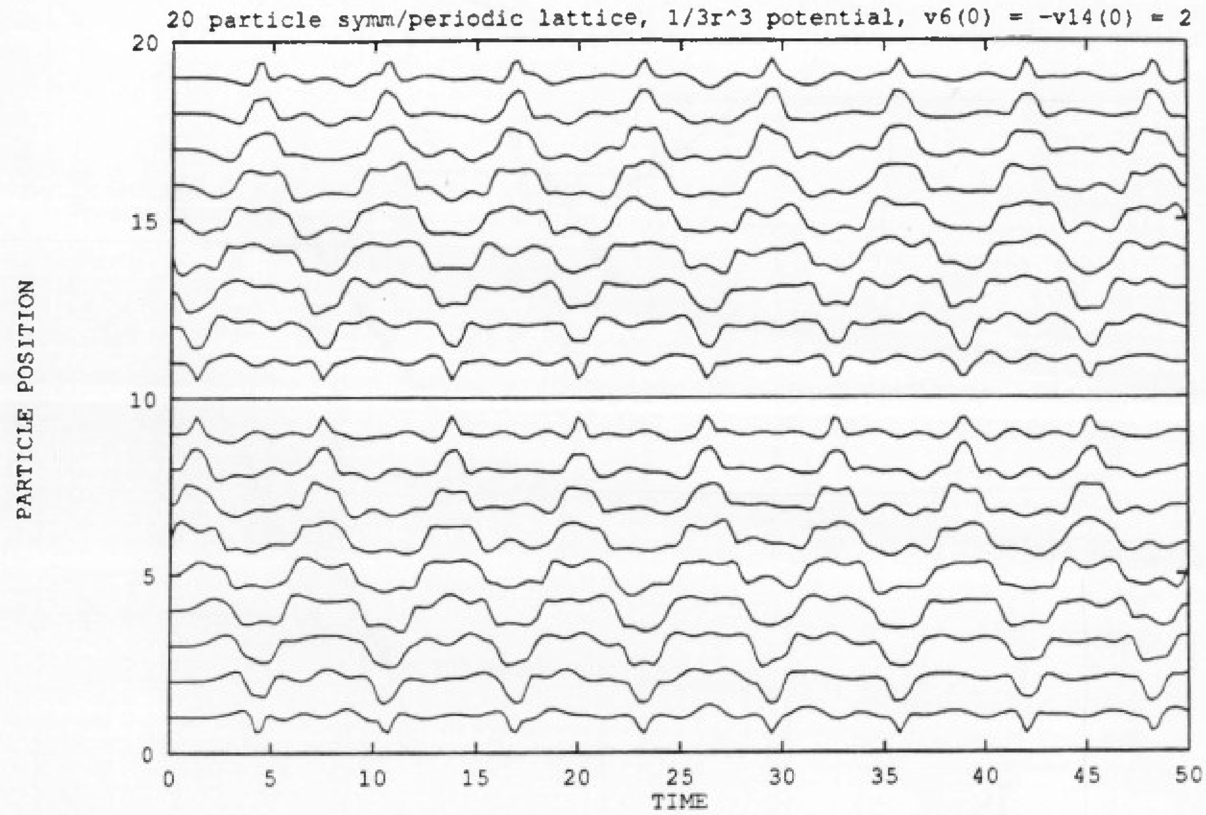
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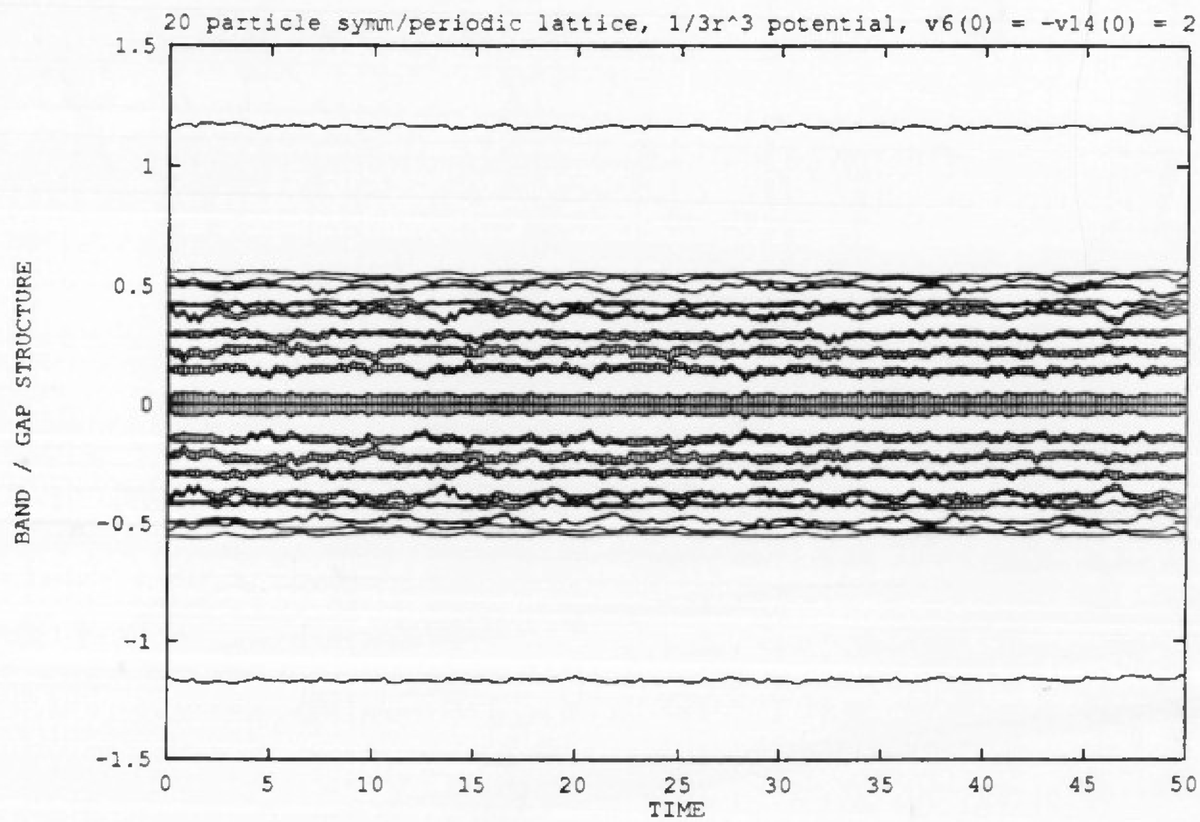
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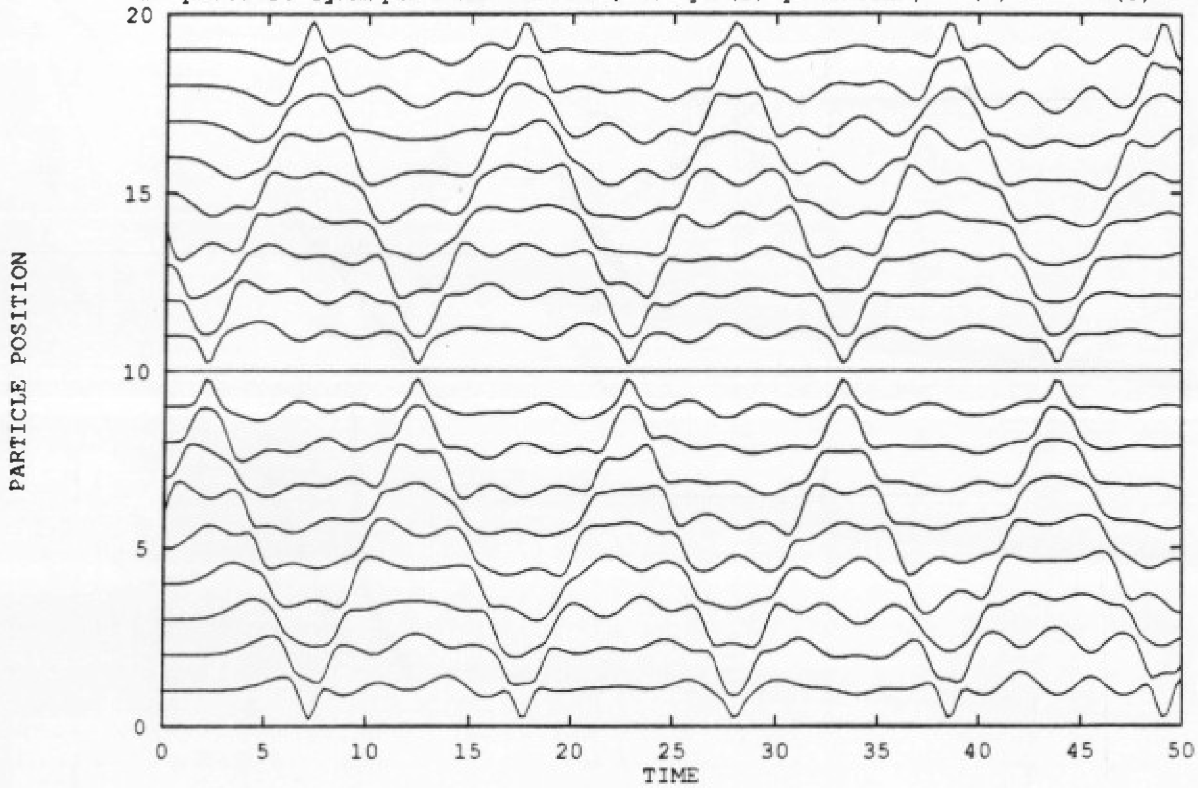

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